RC CIRCUIT AS A FILTERING AND PHASE SHIFTING NETWORK

OBJECTIVES:

(I) **S**tudy the transfer function and phase shift of a low pass RC filter network.
(II) Study the transfer function and phase shift of a high pass RC filter network.

OVERVIEW:

Filter circuits are used in a wide variety of applications. In the field of telecommunication, band-pass filters are used in the audio frequency range (20 Hz to 20 kHz) for modems and speech processing. High-frequency band-pass filters (several hundred MHz) are used for channel selection in telephone central offices. Data acquisition systems usually require anti-aliasing low-pass filters as well as low-pass noise filters in their preceding signal conditioning stages. System power supplies often use band-rejection filters to suppress the 50-Hz line frequency and high frequency transients.

Frequency-selective or filter circuits pass only those input signals to the output that are in a desired range of frequencies (called pass band). The amplitude of signals outside this range of frequencies (called stop band) is reduced (ideally reduced to zero). The frequency between pass and stop bands is called the cut-off frequency (ω_c). Typically in these circuits, the input and output currents are kept to a small value and as such, the current transfer function is not an important parameter. The main parameter is the voltage transfer function in the frequency domain, $H_v(j\omega) = V_o/V_i$. Subscript v of Hv is frequently dropped. As $H(j\omega)$ is complex number, it has both a magnitude and a phase, filters in general introduce a phase difference between input and output signals.

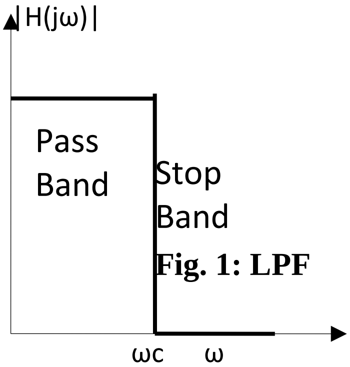


Fig. 1: LPF

LOW AND HIGH-PASS FILTERS

A low pass filter or LPF attenuates or rejects all high frequency signals and passes only low frequency signals below its characteristic frequency called as cut-off frequency, ω_c. An ideal low-pass fillter's transfer function is shown in Fig. 1. A high pass filter or HPF, is the exact opposite of the LPF circuit. It attenuates or rejects all low frequency signals and passes only high frequency signals above ω_c.

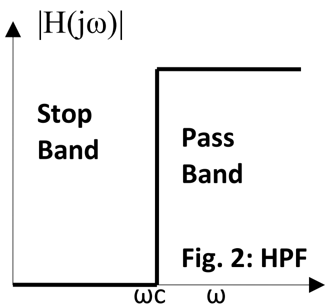


Fig. 2: HPF

In practical filters, pass and stop bands are not clearly defined, $|H(j\omega)|$ varies continuously from its maximum towards zero. The cut-off frequency is, therefore, defined as the frequency at which $|H(j\omega)|$ is reduced to $1/\sqrt{2}$ or 0.7 of its maximum value. This corresponds to signal power being reduced by 1/2 as $P \propto V^2$.

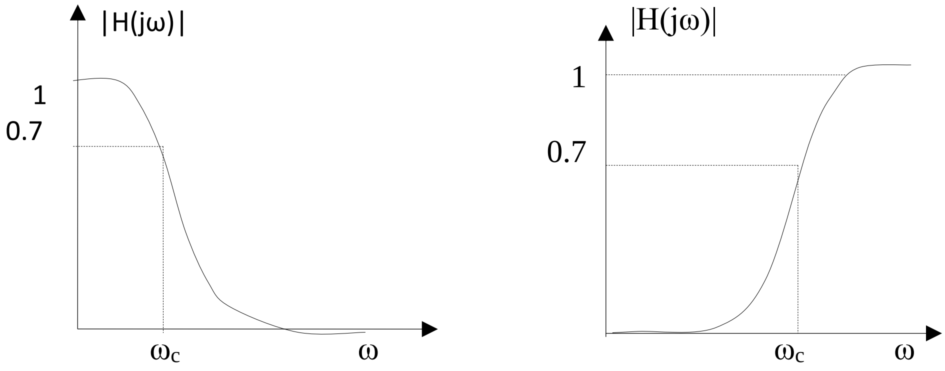


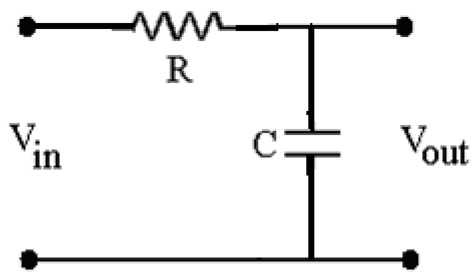
Fig.3: Transfer functions of practical low and high pass filter

RC Filter:

The simplest passive filter circuit can be made by connecting together a single resistor and a single capacitor in series across an input signal, (Vin) with the output signal, (Vout) taken from the junction of these two components. Depending on which way around we connect the resistor and the capacitor with regards to the output signal determines the type of filter construction resulting in either a Low Pass or a High Pass Filter. As there are two passive components within this type of filter design the output signal has amplitude smaller than its corresponding input signal, therefore passive RC filters attenuate the signal and have a gain of less than one, (unity).

Low-pass RC Filter

A series RC circuit as shown also acts as a low-pass filter. For no load resistance (output is open circuit, R → ∞):



$V_0 = \frac{1/(j\omega C)}{R+(1/j\omega C)} V_i = \frac{1}{1+j(\omega \text{RC})} V_i H(j\omega) = \frac{V_0}{V_i} = \frac{1}{1+j\omega \text{RC}}$

To find the cut-off frequency (ω_c), we note

$$|H(j\omega)| = \frac{1}{\sqrt{1+}}$$

When ω→0, |H(jω)| is maximum and →1.

For $\omega = \omega_c$, $|H(j\omega_c)| = 1/\sqrt{2}$. Thus

$$|H(j\omega_c)| = \frac{1}{\sqrt{1+}}$$

Fig.4: Low pass RC filter circuit

Input Impedance:

$$Z_i = R + \frac{1}{j\omega C} \text{and} |Z_i| = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

The value of the input impedance depends on the frequency ω. For good voltage coupling, we need to ensure that the input impedance of this filter is much larger than the output impedance of the previous stage. Thus, the minimum value of Z_i is an important number. Z_i is minimum when the impedance of the capacitor is zero ($\omega \to \infty$), i.e. $Z_i|_{min} = R$.

Output Impedance:

The output impedance can be found by shorting the source and finding the equivalent impedance between output terminals:

$$Z_0 = R || \frac{1}{j\omega C}$$

where the source resistance is ignored. Again, the value of the output impedance also depends on the frequency ω. For good voltage coupling, we need to ensure that the output impedance of this filter is much smaller than the input impedance of the next stage. The maximum value of Z_0 is an important number. Z_0 is maximum when the impedance of the capacitor is ∞ ($\omega \to 0$), i.e. $Z_0|_{max} = R$.

Bode Plots and Decibel

The ratio of output to input power in a two-port network is usually expressed in Bell:

$$\text{Number of Bels } = \log_{10}\left(\frac{P_0}{P_i}\right) = 2\log_{10}\left(\frac{V_0}{V_i}\right)$$

Bel is a large unit and decibel (dB) is usually used:

$$\text{Number of decibels } = 10\log_{10}\left(\frac{P_0}{P_i}\right) = 20\log_{10}\left(\frac{V_0}{V_i}\right)$$

There are several reasons why decibel notation is used:

1) Historically, the analog systems were developed first for audio equipment. Human ear ‘hears" the sound in a logarithmic fashion. A sound which appears to be twice as loud actually has 10 times power, etc. Decibel translates the output signal to what ear hears.

2) If several two-port network are placed in a cascade (output of one is attached to the input of the next), it is easy to show that the overall transfer function, H, is equal to the product of all transfer functions:

$$|H(j\omega)| = |H_1(j\omega)| \times |H_2(j\omega)| \times \dots$$
$$20\log_{10}|H(j\omega)| = 20\log_{10}|H_1(j\omega)| + 20\log_{10}|H_2(j\omega)| + \dots$$
$$|H(j\omega)|_{\text{dB}} = |H_1(j\omega)|_{\text{dB}} + |H_2(j\omega)|_{\text{dB}} + \dots$$

making it easier to understand the overall response of the system.

3) Plot of $|H(j\omega)|_{dB}$ versus frequency has special properties that again makes analysis simpler as is seen below.

For example, using dB definition, we see that, there is 3 dB difference between maximum gain and gain at the cut-off frequency:

$$20\log_{10}|H(j\omega_c)| - 20\log_{10}|H(j\omega)|_{\max} = 20\log_{10}\frac{|H(j\omega_c)|}{|H(j\omega)|_{\max}} = 20\log\left(\frac{1}{\sqrt{2}}\right) = -3\text{dB}$$

Bode plots are plots of magnitude in dB and phase of H(jω) versus frequency in a semi-log format. Bode plots of first-order low-pass filters (include one capacitor) display the following typical characteristics:

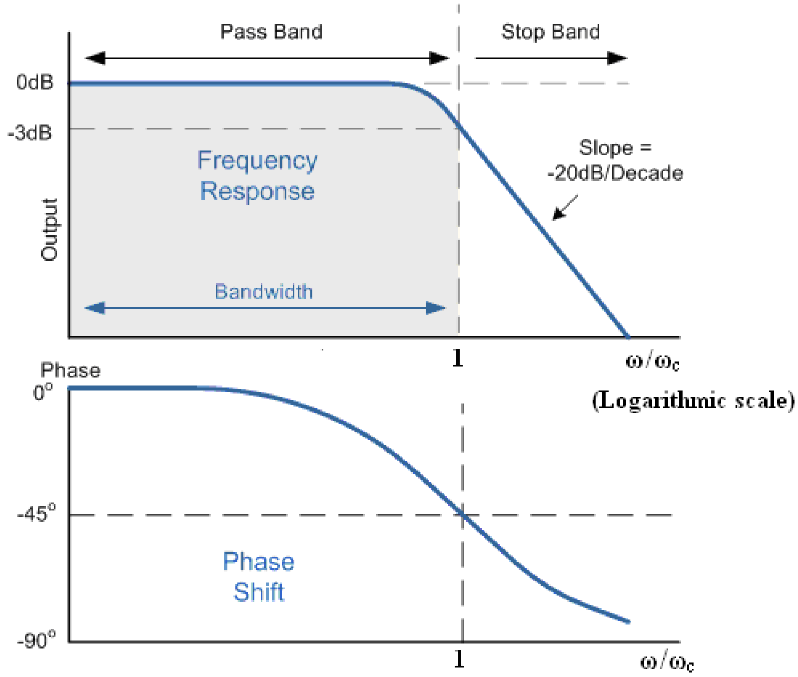


Fig.5: Bode Plots for low-pass RC filter

At high frequencies, $\omega/\omega_c >> 1$, $|H(j\omega)| \approx 1/(\omega/\omega_c)$ and $|H(j\omega)|_{dB} = 20 \log (\omega_c) - 20 \log \omega$, which is a straight line with a slope of -20 dB/decade in the Bode plot. It means that if ω is increased by a factor of 10 (a decade), $|H(j\omega)|_{dB}$ changes by -20 dB.

At low frequencies $\omega/\omega_c << 1$, $|H(j\omega)| \approx 1$, which is also a straight line in the Bode plot. The intersection of these two “asymptotic" values is at $1 = 1/(\omega/\omega_c)$ or $\omega = \omega_c$. Because of this, the cut-off frequency is also called the “corner" frequency.

The behavior of the phase of H(jω) can be found by examining $\varphi = -\tan^{-1}\left(\frac{\omega}{\omega_c}\right)$. At high frequencies, $\omega/\omega_c >> 1, \varphi \approx -90^0$ and at low frequencies, $\omega/\omega_c << 1, \varphi \approx 0$. At cut-off frequency, $\varphi \approx -45^0$.

High-pass RC Filter

A series RC circuit as shown acts as a high-pass filter. For no load resistance (output open circuit), we have:

$$V_0 = \frac{R}{R + (1/j\omega C)} V_i = \frac{1}{1 - j(1/\omega \,\mathrm{RC})} V_i$$

$$H(j\omega) = \frac{V_0}{V_i} = \frac{1}{1 - j(1/\omega \,\mathrm{RC})}$$

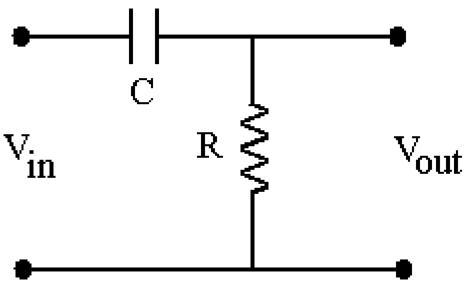


Fig.6: High pass RC filter circuit

The gain of this filter, |H(jω)| is maximum is maximum when denominator is smallest, i.e., $\omega \rightarrow \infty$, leading to $|H(j\omega)|_{max} = 1$. Then, the cut-off frequency can be found as

$$|H(j\omega_c)| = \frac{1}{\sqrt{1 +}}$$

Input and output impedances of this filter can be found similar to the procedure used for low-pass filters:

Input impedance: $Z_i = R + \frac{1}{j\omega C}$ and $Z_{i\ \text{min}} = R$

Output Impedance: $Z_0 = R || \frac{1}{j\omega C}$ and $Z_{0\ \text{max}} = R$

Bode Plots of first-order high-pass filters display the following typical characteristics:
At low frequencies, $\omega/\omega_c << 1$, $|H(j\omega)| \alpha\ \omega$ (a +20dB/decade line) and $\varphi \approx 90^0$.
At high frequencies, $\omega/\omega_c >> 1$, $|H(j\omega)| \approx 1$ (a line with a slope of 0) and $\varphi \approx 0^0$.

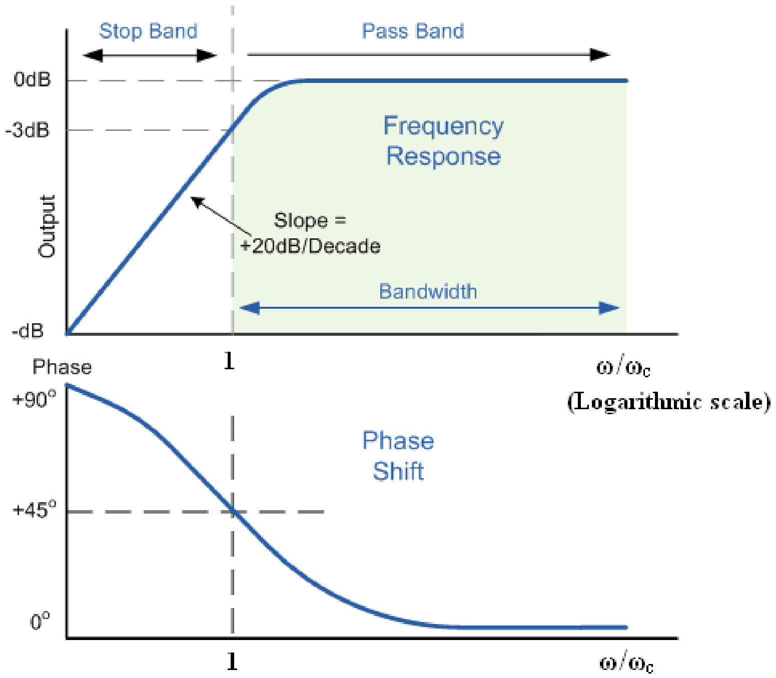
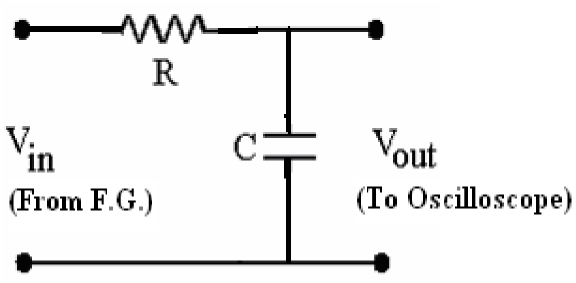


Fig.7: Bode Plots for high-pass RC filter

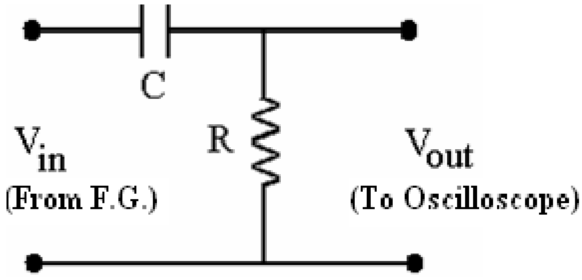
Circuit Components/Instruments:

(i) Resistor (5.6 kΩ), (ii) Capacitor (10 kpF), (iii) Function generator, (iv) Oscilloscope, (v) Connecting wires, (vi) Breadboard

Circuit Diagrams:



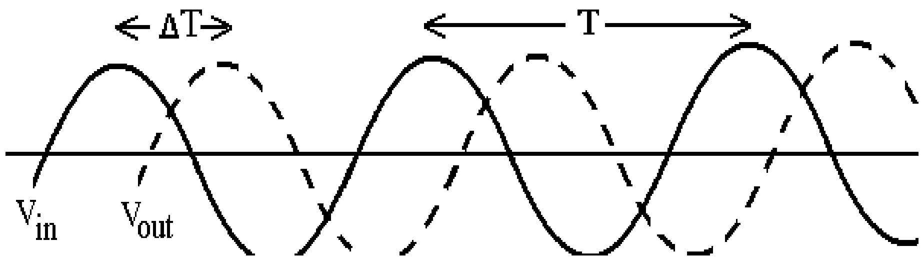
(Low pass RC filter)



(High pass RC filter)

Procedure:

1. Begin lab by familiarizing yourself with the function generator and oscilloscope.
2. Read and also measure the values of R and C.
3. Using the scope set the function generator to produce a 10 V(pp) sine wave. This signal will be used for the input. Do not change the amplitude of this signal during the experiment.
4. Set up the low/high pass RC filter on the breadboard as shown in the circuit diagram. Use the function generator to apply a 10 V(pp) sine wave signal to the input. Use the dual trace oscilloscope to look at both V_{in} and V_{out}. Be sure that the two oscilloscope probes have their grounds connected to the function generator ground.
   For several frequencies between 20 Hz and 20 kHz (the audio frequency range) measure the peak-to-peak amplitude of V_{out} and phase differenceφ. You could also calculate phase shift by measuring lead/lag time,ΔT, as shown in the diagram below using the expression, $\varphi_R(\text{deg}) = \left(\frac{\Delta T}{T}\right) \times 360^0$.
5. Check often to see that V_{in} remains roughly at the set value. Take enough data (at least up to 10 times the cut-off frequency, for low pass and down to 1/10 times cut-off frequency, for high pass filter) so as to make your analysis complete. If needed use the STOP button of oscilloscope at a desired frequency to acquire data.



6. From your measurements determine the ratio
   $$|H(j\omega)| = \left|\frac{V_o}{V_i}\right| = \frac{V_o(\text{pp})}{V_i(\text{pp})}$$
   and compute this ratio by using the formula
   $|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}}$, for low pass filter and
   $|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega_c}{\omega}\right)^2}}$, for high pass filter

Observations:

R = ______________, C = ____________

(I) For Low Pass Filter: $V_{in}(pp)$ = _______, ω_c= 1/RC = ________, f_c = ______

(a) Table for |H(jω)|:

Sl. No.	Frequency, f (kHz) ($\omega = 2\pi f$)	$\frac{\omega}{\omega_c}$	V_0(pp) (Volt)	$\|H(j\omega)\| = \frac{V_o(\text{pp})}{V_i(\text{pp})}$	$\|H(j\omega)\|_{dB}$	$\|H(j\omega)\| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}}$
1	0.02					
2	..					
..	..					
..	..					

(b) Table for phase angle φ:

Sl. No.	Frequency, f (kHz) ($\omega = 2\pi f$)	$\frac{\omega}{\omega_c}$	ΔT (ms)	T (ms)	$\varphi = \left(\frac{\Delta T}{T}\right) \times 360^0$ (deg)	$\varphi = -\tan^{-1}\left(\frac{\omega}{\omega_c}\right)$ (deg)
1	0.02					
2	..					
..	..					
..	..					

(II) For High Pass Filter: $V_{in}(pp)$ = _______, ω_c= **1/RC** = ________, f_c = ______

(c) Table for |H(jω)|:

Sl. No.	Frequency, f (kHz) ($\omega = 2\pi f$)	$\frac{\omega_c}{\omega}$	V_0(pp) (Volt)	$\|H(j\omega)\| = \frac{V_o(\text{pp})}{V_i(\text{pp})}$	$\|H(j\omega)\|_{dB}$	$\|H(j\omega)\| = \frac{1}{\sqrt{1+\left(\frac{\omega_c}{\omega}\right)^2}}$
1	0.02					
2	..					
..	..					
..	..					

(d) Table for phase angle φ:

Sl. No.	Frequency, f (kHz) ($\omega = 2\pi f$)	$\frac{\omega_c}{\omega}$	**ΔT (ms)**	**T (ms)**	$\varphi = \left(\frac{\Delta T}{T}\right) \times 360^0$ **(deg)**	$\varphi = -\tan^{-1}\left(\frac{\omega_c}{\omega}\right)$ **(deg)**
1	0.02					
2	..					
..	..					
..	..					

Graphs: Trace and study bode plots of $|H(j\omega)|_{dB}$ and φ versus $f(\times 2\pi)$ in a semi-log format for low/high pass RC filter. Determine the cut-off frequency from graph. Also, estimate the frequency roll-off for each filter.

Discussions:

Precautions: